

# PERIYAR UNIVERSITY

Re-accredited with 'A' grade by the NAAC

PERIYAR PALKALAI NAGAR

SALEM - 11



## **M.Sc. Branch – I (B): Mathematics**

**(SEMESTER PATTERN)**

**(Under Choice Based Credit System)**

**(For University Department)**

**&**

**(PG Center, Dharmapuri)**

## **REGULATIONS AND SYLLABUS**

**(For candidates admitted from 2018-2019 onwards)**

**1. OBJECTIVES:**

- ❖ To have well-rounded introduction to the main branches of modern Mathematics that keeps an appropriate balance between Theoretical and Applied Mathematics.
- ❖ To have training and knowledge necessary to appreciate and apply Modern Mathematics and to find rewarding careers in Mathematics related fields, or to pursue research in Mathematics.
- ❖ To have an understanding of the tools required to be able to quantitatively analyze and understand the natural and social world,
- ❖ To have the ability to solve problems, think analytically, and reason quantitatively.
- ❖ To have the ability to access and communicate Mathematical information.
- ❖ To have an appreciation of the concept of formal Mathematical proof and the ability to write simple proofs.
- ❖ To participate actively in Mathematics related events namely Conferences / Seminars /Workshops and Quiz programs.

**2. DURATION OF THE PROGRAMME**

The two-year postgraduate programme in M.Sc. Mathematics consists of four semesters under Choice Based Credit System.

**3. ELIGIBILITY**

A candidate who has passed B.Sc. Degree Examination in Branch I- Mathematics and Mathematics (CA) of this University or an examination of some other university accepted by the syndicate as equivalent there to shall be permitted to appear and qualify for the M.Sc. Mathematics (CBCS) Degree Examination of this university after a course of two academic years in the Department of Mathematics of Periyar University.

#### **4. DISTRIBUTION OF CREDIT POINTS AND MARKS**

The minimum credit requirement for a two year Master's programme shall be **90 Credits**. The break-up of credits for the programme is as follows:

- ✚ Core Courses : Minimum 62 credits
  - ✚ Elective Courses : Minimum 16 credits
  - ✚ Supportive Courses : Minimum 06 credits
  - ✚ Project : 04 credits
  - ✚ Human Rights : 02 credits
  - ✚ SWAYAM/MOOC/NPTEL : 08 credits
- *Human Rights will be a compulsory course with 2 credits offered in the II-semester as add on course.*
  - *SWAYAM/MOOC/NPTEL online Courses will be offered in the II & III semesters with 4 credits each as add on courses.*

#### **4. COURSE OF STUDY**

The courses of study for the degree shall be in Branch I (B) - Mathematics (Choice Based Credit System) with internal assessment according to syllabi prescribed from time to time. The **Internal Assessment** is distributed to tests, seminar, assignment and attendance as **10, 05, 05** and **05** marks, respectively.

	Marks			No. of Subjects	Total Marks	Credits
	External	Internal	Total			
<b>For Each Paper</b>	<b>75</b>	<b>25</b>	<b>100</b>	<b>22</b>	<b>2200</b>	<b>96</b>
<b>Dissertation + Viva Voce</b>	<b>25+25</b>	<b>25+25</b>	<b>100</b>	<b>01</b>	<b>100</b>	<b>04</b>
	<b>Grand Total</b>			<b>23</b>	<b>2300</b>	<b>100</b>

Dissertation : **100** (Internal Valuation 25 + External Valuation 25 and Joint Viva Voce 25 + 25 Marks]

## **5. STRUCTURE OF THE COURSE**

S. No	COURSE CODE	TITLE OF THE COURSE	CREDITS	MARKS
<b>I SEMESTER</b>				
1.	18UPMAT1C01	LINEAR ALGEBRA	5	100
2.	18UPMAT1C02	REAL ANALYSIS -I	5	100
3.	18UPMAT1C03	ORDINARY DIFFERENTIAL EQUATIONS	5	100
4.	18UPMAT1C04	MECHANICS	5	100
5.		ELECTIVE COURSE - I	4	100
<b>II SEMESTER</b>				
6.	18UPMAT1C05	ABSTRACT ALGEBRA	5	100
7.	18UPMAT1C06	REAL ANALYSIS - II	5	100
8.	18UPMAT1C07	PARTIAL DIFFERENTIAL EQUATIONS	5	100
9.		ELECTIVE COURSE - II	4	100
10.		SUPPORTIVE COURSE - I	3	100
11.	06PHR01	HUMAN RIGHTS	2	100
12.		SWAYAM - I	4	100
<b>III SEMESTER</b>				
13.				
14.				
15.				
16.				
17.				
18.				
<b>V SEMESTER</b>				
19.				
20.				
21.				
22.				
23.				

## **ELECTIVE COURSES**

<b>S.No</b>	<b>COURSE CODE</b>	<b>TITLE OF THE COURSE</b>	<b>CREDITS</b>
1.	18UPMAT1E01	Discrete Mathematics	4
2.	18UPMAT1E02	Analytic Number Theory	4
3.	18UPMAT1E03	Difference Equations	4
4.	18UPMAT1E04	Numerical Analysis	4
5.	18UPMAT1E05	Methods of Applied Mathematics	4
6.	18UPMAT1E06	Optimization Techniques	4
7.	18UPMAT1E07	Combinatorial Mathematics	4
8.	18UPMAT1E08	Fuzzy Sets and their Applications	4
9.	18UPMAT1E09	Representation Theory of Finite Groups	4
10.	18UPMAT1E10	Non Commutative Algebra – I	4
11.	18UPMAT1E11	Non Commutative Algebra – II	4
12.	18UPMAT1E12	Commutative Algebra	4
13.	18UPMAT1E13	Control Theory	4
14.	18UPMAT1E14	Stochastic Differential Equations	4
15.	18UPMAT1E15	Number Theory	4
16.	18UPMAT1E16	Differential Geometry	4
17.	18UPMAT1E17	Advanced Partial Differential Equations	4
18.	18UPMAT1E18	Nonlinear Differential Equations	4
19.	18UPMAT1E19	Mathematical Biology	4
20.	18UPMAT1E20	Fluid Dynamics	4

## **SUPPORTIVE COURSES**

<b>S.No</b>	<b>COURSE CODE</b>	<b>TITLE OF THE COURSE</b>	<b>CREDITS</b>
1.	18UPMAT1S01	Applied Mathematics – I	4
2.	18UPMAT1S02	Applied Mathematics – II	4
3.	18UPMAT1S03	Numerical & Statistical Methods	4
4.	18UPMAT1S04	Discrete Mathematics	4

## **6. EXAMINATION**

For the purpose of uniformity, particularly for interdepartmental transfer of credits, there shall be a uniform pattern of examination to be adopted by all the teachers offering courses. There shall be three tests, one seminar and one assignment for internal evaluation and End semester examination during each semester.

The distribution of marks for internal evaluation and End Semester Examination shall be 25 marks and 75 marks, respectively. Further, distribution of internal marks shall be 10 marks for test, 5 marks for seminar, 5 marks for assignment and 5 marks for attendance, respectively. The average of the highest two test marks out of the three internal tests should be taken for Internal Assessment.

## **7. QUESTION PAPER PATTERN**

### **(a) Question paper pattern for Theory Examination**

**Time:** 3 Hours

**Maximum Marks:** 75

#### **Part – A (20 X 1 = 20 Marks)**

Objective Type questions

- Answer **ALL** Questions

(Four questions from each unit)

#### **Part – B (3 X 5 = 15 Marks)**

Analytical Type questions (Problems only)

- Answer any **THREE** questions out of **FIVE** questions

(One question from each unit)

#### **Part – C (5 X 8 = 40 Marks)**

Descriptive Type questions

- Answer **ALL** Questions

(Two questions from each unit with internal choice)

### **(b) Question paper pattern for Practical Examination**

Time: 3 Hours

Maximum: **100** (Internal: 40 + External: 60) Marks

#### **The components of 40 marks are**

Periodical assessment	- 20 marks
Test (best 2 out of 3)	- 10 marks
Record	- 10 marks

#### **The components of 60 marks are**

Experiments	- 40 marks
Viva-voce	- 10 marks
Record	- 10marks

**Passing Minimum : 30 Marks** (Aggregate of Experiments, Viva-voce and Record)  
(No passing minimum for records)

There will be one question with or without subsections to be asked for the practical examination. Every question should be chosen from the question bank prepared by the examiner(s). A question may be used for at most three students in a batch.

#### **8. PASSING MINIMUM**

A candidate who has secured a minimum of 50% marks in all the courses (including practical) prescribed in the programme and earned a minimum of **90 credits** will be considered to have passed the Master's programme.

For the Practical paper, a minimum of 30 marks out of 60 marks in the University examination and the record notebook taken together is necessary for a pass. There is no passing minimum for the record notebook. However submission of record notebook is a must.

For the Project work and viva-voce, a candidate should secure 50% of the marks for pass. The candidate should attend viva-voce examination to secure a pass in the Project.

#### **9. COMMENCEMENT OF THIS REGULATION**

These regulations shall take effect from the academic year 2018-19, that is, for students who are admitted to the first year of the programme during the academic year 2018-19 and thereafter.

#### **10. PROJECT AND EDUCATIONAL TOUR:**

For M.Sc Mathematics students, the project is individual and compulsory. In order to choose their topics/titles for the project, the students may like to visit the Libraries at the Universities / Indian Institute of Technology / Institute of Mathematical Sciences etc. So, the Department of Mathematics may arrange an Educational Tour either at the end of III semester or in the beginning of IV semester, for the students to visit the Libraries.

**(a) Dissertation Topic:**

The topic of the dissertation shall be assigned to the candidate at the beginning of third semester and a copy of the same should be submitted to the University for approval.

**(b) No. of copies of dissertation:**

Students should prepare three copies of dissertation and submit the same for the evaluation by Examiners. After evaluation one copy is to be retained in the University Library, one in the Department Library and one with the student.

**(c) Format for the preparation of the dissertation:**

- (a) Title page
- (b) Bonafide Certificate
- (c) Acknowledgement
- (d) Table of contents

**CONTENTS**

<b>Chapter No.</b>	<b>Title</b>	<b>Page No.</b>
1.	Introduction	
2.	Review of Literature	
3.	Summary	
4.	Results	
5.	References	



**Format of the Title Page**

**TITLE OF THE DISSERTATION**

Dissertation submitted in partial fulfillment of the requirements for the award of the

Degree of Master of Science in

**MATHEMATICS**

**(Under Choice Based Credit System)**

Submitted to

Department of Mathematics

Periyar University, Salem – 636 011.

By

Students Name :

Register Number :

Department :

Year :

## Format of the Certificate

### CERTIFICATE

This is to certify that the dissertation entitled .....submitted in partial fulfillment of the requirements for the award of the Degree of Master of Science in **MATHEMATICS (Under Choice Based Credit System)** to the Periyar University, Periyar Palkalai Nagar, Salem is a record of bonafide research work carried out by ..... under my supervision and guidance and that no part of the dissertation has been submitted for the award of any degree, diploma, fellowship or other similar titles or prizes and that the work has not been published in part or full in any scientific or popular journals or magazines.

Date:

Place:

Signature of the Guide

Signature of the Head of the Department

18UPMAT1C01	LINEAR ALGEBRA	L	T	P	C
		4	1	0	5

**OBJECTIVE:**

The objective of this course is to develop a strong foundation in linear algebra that provide a basic for advanced studies not only in mathematics but also in other branches like engineering, physics and computers, etc. Particular attention is given to canonical forms of linear transformations, diagonalizations of linear transformations, matrices and determinants.

**UNIT I: Linear transformations**

Linear transformations – Isomorphism of vector spaces – Representations of linear transformations by matrices – Linear functionals.

**UNIT II: Algebra of polynomials**

The algebra of polynomials –Polynomial ideals - The prime factorization of a polynomial - Determinant functions.

**UNIT III: Determinants**

Permutations and the uniqueness of determinants – Classical adjoint of a (square) matrix – Inverse of an invertible matrix using determinants – Characteristic values – Annihilating polynomials.

**UNIT IV: Diagonalization**

Invariant subspaces – Simultaneous triangulations – Simultaneous diagonalization – Direct-sum decompositions – Invariant direct sums – Primary decomposition theorem.

**UNIT V: The Rational and Jordan forms**

Cyclic subspaces – Cyclic decompositions theorem (Statement only) – Generalized Cayley – Hamilton theorem - Rational forms – Jordan forms.

**TEXT BOOK:**

**Kenneth M Hoffman** and **Ray Kunze**, Linear Algebra, 2<sup>nd</sup> Edition, Prentice-Hall of India Pvt. Ltd, New Delhi, 2013.

UNIT	Chapter(s)	Sections
I	3	3.1 – 3.5
II	4 & 5	4.1, 4.2, 4.4, 4.5 and 5.1, 5.2
III	5 & 6	5.3, 5.4 and 6.1 – 6.3
IV	6	6.4 – 6.8
V	7	7.1 – 7.3

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **M. Artin**, “Algebra”, Prentice Hall of India Pvt. Ltd., 2005.
2. **S.H. Friedberg, A.J. Insel and L.E Spence**, “Linear Algebra”, 4<sup>th</sup> Edition, Pritice-Hall of India Pvt. Ltd., 2009.
3. **I.N. Herstein**, “Topics in Algebra”, 2<sup>nd</sup> Edition, Wiley Eastern Ltd, New Delhi, 2013.
4. **J.J. Rotman**, “Advanced Modern Algebra”, 2<sup>nd</sup> Edition, Graduate Studies in Mathematics, Vol. 114, AMS, Providence, Rhode Island, 2010.
5. **G. Strang**, “Introduction to Linear Algebra”, 2<sup>nd</sup> Edition, Prentice Hall of India Pvt. Ltd, 2013.

**LEARNING OUTCOMES:** At the end of the course, students will be able

- to describe a diagonalizable operator  $T$  in a language of invariant direct sum decompositions (projections which commute with  $T$ ).
- to find the minimal polynomials, Jordan forms and the rational forms of real matrices.

18UPMAT1C02	REAL ANALYSIS - I	L	T	P	C
		4	1	0	5

**OBJECTIVE:**

The course will develop a deeper and more rigorous understanding of calculus including defining terms and proving theorems about functions, sequences, series, limits, continuity and derivatives. The course will develop specialized techniques in problem solving.

**UNIT I: Basic Topology**

Finite, Countable and Uncountable Sets – Metric Spaces – Compact Sets – Connected Sets (Perfect sets - Omitted).

**Unit II: Numerical Sequences and Series**

Convergent sequences – Subsequences – Cauchy sequences - Upper and lower limits - Some special sequences – Series – Series of nonnegative terms - The number  $e$  - The root and ratio tests.

**Unit III: Rearrangements of Series**

Power series - Summation by parts - Absolute convergence - Addition and multiplication of series – Rearrangements.

**UNIT III: Continuity**

Limit of Functions – Continuous functions - Continuity and Compactness – Continuity and Connectedness – Discontinuities – Monotonic functions – Infinite limits and Limits at infinity.

**UNIT IV: Differentiation**

The derivative of a real function – Mean value theorems – The continuity of the Derivative – L' Hospital's Rule – Derivatives of Higher order – Taylor's theorem – Differentiation of Vector-valued functions.

**TEXT BOOK:**

**Walter Rudin**, "*Principles of Mathematical Analysis*", 3<sup>rd</sup> Edition, McGraw Hill Book Co., Kogaskusha, 1976.

UNIT	Chapter(s)	Pages
I	2	24 - 40, 42 - 46
II	3	47 - 68
III	3	69 - 82
IV	4	83 - 102
V	5	103 - 119

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **Tom M. Apostol**, “*Mathematical Analysis*”, Narosa Publishers, New Delhi, 2002.
2. **R. G. Bartle and D.R. Sherbert**, “*Introduction to Real Analysis*”, John Wiley & Sons, New York, 1982.
3. **W.J. Kaczor and M.T. Nowak**, “*Problems in Mathematical Analysis I – Real Numbers , Sequences and Series*”, American Mathematical Society, 2000.
4. **W.J. Kaczor and M.T. Nowak**, “*Problems in Mathematical Analysis II – Continuity and Differentiation*”, American Mathematical Society, 2000.
5. **Steven G. Krantz**, *Real Analysis and Foundations*, 4<sup>th</sup> Edition, CRC Press, 2017.
6. **H.H.Sohrab**, “*Basic Real Analysis*”, Springer International Edition, India, 2006.

**LEARNING OUTCOMES:**

On successful completion of this course, students will be able

- to give the definition of concepts related to metric spaces, such as continuity, compactness, completeness and connectedness that will help for further studies within topology and functional analysis.
- to demonstrate an understanding of limits and how they are used in sequences, series, continuity and differentiation.
- to construct rigorous mathematical proofs of basic results in real analysis.

<b>18UPMAT1C03</b>	<b>ORDINARY DIFFERENTIAL EQUATIONS</b>	L	T	P	C
		4	1	0	5

**OBJECTIVE:**

The objective of this course is to equip the students with knowledge of some advanced concepts related to ordinary differential equations and to understand the concepts related to the solution of ordinary differential equations.

**UNIT I: Linear Equations with Constant Coefficients**

The second order homogeneous equation - Initial value problems for second order equations - Linear dependence and independence – A formula for Wronskian.

**UNIT II:**

**Linear Equations with Constant Coefficients:** The non-homogeneous equation of order two – The homogeneous equation of order n -- A special method for solving the non-homogeneous equation.

**Linear Equations with Variable Coefficients:** Reduction of the order of a homogeneous equation – The Legendre Equation.

**UNIT III: Linear Equations with Regular Singular Points**

The Euler equation - Second order equations with regular singular points – The Bessel Equation - The Bessel Equation (continued).

**UNIT IV: Existence and Uniqueness of Solutions to First Order Equations**

Equations with variables separated - Exact equations – The method of successive approximations  
 – The Lipschitz condition- Convergence of the successive approximations

**UNIT V: Boundary Value Problems**

Sturm-Liouville problem – Green’s functions.

**TEXT BOOK:**

1. **Earl A. Coddington**, “*An Introduction to Ordinary Differential Equations*”, Prentice Hall of India, New Delhi, 2011.
2. **S. G. Deo, V. Lakshmikantham and V. Raghavendra**, “*Textbook of Ordinary Differential Equations*”, Tata McGraw-Hill, New Delhi, 1997.

UNIT	Chapters	Sections
I	2 of [1]	1 – 5
II	2 of [1]	6, 7, 11
	3 of [1]	5, 8
III	4 of [1]	1, 2, 3, 7, 8
IV	5 of [1]	1- 6
V	7 of [2]	7.1 – 7.3

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES**

1. **R.P. Agarwal and R. C. Gupta**, “*Essentials of Ordinary Differential Equation*”, McGraw Hill, New York, 1991.
2. **A. K. Nandakumaran, P.S. Satti, Raju K. George**, “*Ordinary Differential Equations: Principles and Applications*”, Cambridge University Press, 2017.
3. **D. Rai, D.P. Choudhury and H.I. Freedman**, “*A Course in Ordinary Differential Equations*”, Narosa Publ. House, Chennai, 2004.
4. **Tyn Myint-U**, “*Ordinary Differential Equations*”, Elsevier Science, 1977.
5. **Martin Braun**, “*Differential Equations and Their Applications: An Introduction to Applied Mathematics*”, Springer, 4<sup>th</sup> Edition, 1992.

**LEARNING OUTCOMES:** At the end of the course, students will be able

- to solve the differential equations by using various methods.

18UPMAT1C04	<b>MECHANICS</b>	L	T	P	C
		4	1	0	5

**OBJECTIVE:**

The objective of this course is to understand the Lagrangian and Hamiltonian equations for dynamical systems.

**UNIT I: Mechanical Systems**

The Mechanical system – Generalized coordinates – Constraints – Virtual work – Energy and Momentum.

**UNIT II : Lagrange's Equations**

Derivation of Lagrange's Equations – Examples – Integrals of the motion.

**UNIT III: Hamilton's Equations**

Hamilton's Principle – Hamilton's Equations – Other variational principles.

**UNIT IV: Hamilton – Jacobi Theory**

Hamilton Principle Function – Hamilton-Jacobi Equation – Separability.

**UNIT V: Canonical Transformation**

Differential forms and Generating Functions – Special Transformations – Lagrange and Poisson Brackets.

**TEXT BOOK:**

**D.T. Greenwood**, “*Classical Dynamics*”, Prentice Hall of India, New Delhi, 1985.

UNIT	Chapter	Sections
I	1	1.1 to 1.5
II	2	2.1 to 2.3
III	4	4.1 to 4.3
IV	5	5.1 to 5.3
V	6	6.1 to 6.3

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **H. Goldstein**, “*Classical Mechanics*”, 2<sup>nd</sup> Edition, Narosa Publishing House, New Delhi.
2. **R.D. Gregory**, “*Classical Mechanics*”, Cambridge University Press, 2006
3. **J.L.Synge and B.A.Griffth**, “*Principles of Mechanics*”, 3<sup>rd</sup> Edition, McGraw Hill Book Co., New York, 1970.

**LEARNING OUTCOMES:**

- At the end of the course, the students will understand the formation of differential equations which will help to study the dynamics of mechanical systems.

18UPMAT1C05	ABSTRACT ALGEBRA	L	T	P	C
		4	1	0	5

**OBJECTIVE:**

The objective of this course is to introduce the basic ideas of counting principle, Sylow subgroups, finite abelian groups, field theory and Galois Theory and to see its application to the solvability of polynomial equations by radicals.

**UNIT I: Sylow's Theorem**

Another Counting Principle – 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> parts of Sylow's Theorems – double coset – the normalizer of a group.

**UNIT II: Finite Abelian Groups**

External and Internal direct Products – structure theorem for finite abelian groups – non isomorphic abelian groups - polynomial rings.

**UNIT III: Splitting Field**

Polynomials over rational fields – the Eisenstein criterion - extension fields – roots of polynomials – splitting fields.

**UNIT IV: Galois Theory**

More about roots – simple extension – separable extension – fixed fields – symmetric rational functions – normal extension - Galois group – fundamental theorem of Galois theory.

**UNIT V: Solvability by radicals**

Solvable group – the commutator subgroup – Solvability by radicals - finite fields.

**TEXT BOOK:**

**I.N. Herstein**, Topics in Algebra, 2<sup>nd</sup> Edition, John Wiley and Sons, New York, 1975.

UNIT	Chapter(s)	Sections
I	2	2.11 & 2.12
II	2 & 3	2.13, 2.14, 3.9
III	3 & 5	3.10, 5.1, 5.3
IV	5	5.5 & 5.6
V	5 & 7	5.7, 7.1

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **S. Lang**, "Algebra", 3<sup>rd</sup> Edition, Addison-Wesley, Mass, 1993.
2. **John B. Fraleigh**, "A First Course in Abstract Algebra", Addison Wesley, Mass, 1982.
3. **M. Artin**, "Algebra", Prentice-Hall of India, New Delhi, 1991.
4. **V. K. Khanna and S.K. Bhambri**, "A Course in Abstract Algebra", Vikas Publishing House Pvt Limited, 1993.



**LEARNING OUTCOMES:** At the end of the course, students will be able

- to find the number of Sylow subgroups.
- to find the number of nonisomorphic abelian groups.
- to find the splitting field, Galois group of the given polynomial.
- to check whether the given polynomial is solvable by radicals or not.

<b>18UPMAT1C06</b>	<b>REAL ANALYSIS - II</b>	L	T	P	C
		4	1	0	5

**OBJECTIVE:**

The course will develop a deeper and more rigorous understanding of calculus including defining terms and proving theorems about sequence and series of functions, integration, special functions and multivariable calculus. The course will develop specialized techniques in problem solving.

**UNIT I: Riemann – Stieltjes Integral**

Definition and Existence of the Integral – Properties of the Integral – Integration and Differentiation – Integration of Vector-valued functions – Rectifiable curves.

**UNIT II: Sequences and Series of Functions**

Discussion of main problem – Uniform Convergence - Uniform Convergence and Continuity - Uniform Convergence and Integration – Uniform Convergence and Differentiation.

**Unit III: Sequences and Series of Functions (contd...)**

Equicontinuous families of functions – Stone-Weierstrass Theorems – Algebra of complex valued functions.

**Unit IV: Some special functions**

Power series – The Exponential and Logarithmic functions – Trigonometric Functions – Fourier series - The Gamma functions (Algebraic completeness of the complex field - omitted).

**Unit V: Functions of several variables**

Linear transformations – Differentiation – The contraction principle - The inverse function theorem – The implicit function theorem.

**TEXT BOOK:**

**Walter Rudin**, “*Principles of Mathematical Analysis*”, 3<sup>rd</sup> Edition, McGraw Hill Book Co., Kogaskusha, 1976.

UNIT	Chapter(s)	Pages
<b>I</b>	<b>6</b>	120 – 142
<b>II</b>	<b>7</b>	143 – 154
<b>III</b>	<b>7</b>	155 – 171
<b>IV</b>	<b>8</b>	172 – 203 (Omit Theorem 8.8)
<b>V</b>	<b>9</b>	204 – 228

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **T.M. Apostol**, “*Mathematical Analysis*”, Narosa Publishers, New Delhi, 1985.
2. **W.J.Kaczor and M.T.Nowak**, “*Problems in Mathematical Analysis III - Integration*”, American Mathematical Society, 2000.
3. **A. Browder**, “*Mathematical Analysis, an Introduction*”, Springer-Verlag, New York, 1996.
4. **K.A. Ross**, “*Elementary Analysis: The Theory of Calculus*”, 2<sup>nd</sup> Edition, Springer, New York, 2013.
5. **M. Stoll**, “*Introduction to Real Analysis*”, 2<sup>nd</sup> Edition, Addison-Wesley Longman Inc, 2001.

**LEARNING OUTCOMES:**

On successful completion of this course, students will be able to

- find the integrals of a bounded function on a closed bounded interval
- understand sequences and series of functions and its convergence
- find the derivative of functions of several variables

<b>18UPMAT1C07</b>	<b>PARTIAL DIFFERENTIAL EQUATIONS</b>	L	T	P	C
		4	1	0	5

**OBJECTIVE:**

The objective of this course is to enable the students to understand the concepts related to the solution of partial differential equations arising in various fields.

**UNIT I: Partial differential equations of first order**

Nonlinear partial differential equations of the first order – Cauchy’s method of characteristics – Compatible systems of first order equations – Charpit’s method- Special types of first order equations – Jacobi’s method.

**UNIT II: Partial differential equations of second order**

Linear partial differential equations with constant coefficients – Equations with variable coefficients – The solution of linear hyperbolic equations – Separation of variables – Nonlinear equations of the second order.

**UNIT III: Laplace’s Equation**

Elementary solution of Laplace’s equation – Families of equipotential surfaces – Boundary value problems – Separation of variables – The theory of Green’s function for Laplace equation.

**UNIT IV: The wave equation**

Elementary solutions of the one-dimensional wave equation – Vibrating membranes: Applications of the calculus of variations – Three dimensional problems – Green’s function for the wave equation.

**UNIT V: The Diffusion Equation**

Elementary solutions of the diffusion equation – Separation of variables – The use of Green’s functions.

**TEXT BOOK:**

**I.N. Sneddon**, Elements of Partial Differential Equations, Dover, Singapore, 2006.

UNIT	Chapter	Sections
I	2	7 – 11, 13
II	3	4, 5, 8, 9, 11
III	4	2 – 5, 8
IV	5	2, 4, 5, 7
V	6	3, 4, 6

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **D. Colton**, “*Partial Differential Equations: An Introduction*”, Dover Publishers, New York, 1988.
2. **H. Hattori**, “*Partial Differential Equations: Methods, Applications and Theories*”, World Scientific, Singapore, 2013.
3. **M.D.Raisinghania**, “*Advanced Differential Equations*”, S. Chand & Company, New Delhi, 2013.
4. **K. Sankara Rao**, “*Introduction to Partial Differential Equations*”, Second Edition, Prentice – Hall of India, New Delhi, 2006.

**LEARNING OUTCOMES:** At the end of the course, students will

- be familiar with the modeling assumptions and derivations that lead to PDE’s.
- recognize the major classification of PDEs and the qualitative difference between the classes of equations.
- be competent in solving linear PDEs using classical methods.

18UPMAT1E01	DISCRETE MATHEMATICS	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

The objective of this course is to understand the basic ideas of logic, proof methods and strategy, the growth of functions, counting techniques, pigeonhole principle, recurrence relations, solving recurrences using generating functions, Boolean functions, apply Boolean algebra to circuits and gattng networks, use finite state-machines to model computer operations.

**UNIT I: The Foundation of Logic**

Logic – Propositional equivalence – Predicates and quantifiers – Proof Methods and Strategy – The growth of functions.

**UNIT II: Counting**

Basics of counting – The pigeonhole principle – permutations and combinations – Generalized permutations and combinations – Generating permutations and combinations.

**UNIT III: Advanced counting techniques**

Recurrence relation – Solving recurrence relations – Generating functions.

#### **UNIT IV: Boolean Algebra**

Boolean functions – Representing Boolean functions – Logic Gates – Minimization of circuits.

#### **UNIT V: Modeling Computations**

Finite – state machines with output, finite – State machines with no output – Turing machines

#### **TEXT BOOK:**

**Kenneth H. Rosen, Discrete Mathematics and its Applications**, 7<sup>th</sup> Edition, WCB/ McGraw Hill Publications, New Delhi, 2011.

<b>UNIT</b>	<b>Chapter(s)</b>	<b>Sections</b>
<b>I</b>	<b>1 &amp; 3</b>	<b>1.1 – 1.3, 1.8, 3.2</b>
<b>II</b>	<b>5</b>	<b>5.1 – 5.6</b>
<b>III</b>	<b>6</b>	<b>6.1, 6.2, 6.4</b>
<b>IV</b>	<b>10</b>	<b>10.1 – 10.4</b>
<b>V</b>	<b>12</b>	<b>12.2, 12.3, 12.5</b>

#### **BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **Edward A. Bender and S. Gill Williamson**, “*A Short Course in Discrete Mathematics*”, Dover Publications, 2006.
2. **M.O. Albertson and J.P. Hutchinson**, “*Discrete Mathematics with Algorithms*”, John Wiley & Sons, 2008.
3. **Rajendra Akerkar and Rupali Akarkar**, “*Discrete Mathematics*”, Pearson Education Pvt. Ltd, Singapore, 2004.
4. **J.P. Trembley and R. Manohar**, “*Discrete Mathematical Structures*”, Tata McGraw Hill, New Delhi, 1997.
5. **Martin Aigner**, “*A Course in Enumeration*”, Springer-Verlag, Heidelberg, 2007.
6. **J.H. Van Lint, R.M. Wilson**, “*A Course in Combinatorics*”, 2<sup>nd</sup> Edition, Cambridge University Press, Cambridge, 2001.

#### **LEARNING OUTCOMES:** Students completing this course will be able to

- express a logic sentence in terms of predicates, quantifiers and logical connectives.
  - apply the rules of inference and methods of proof including direct and indirect proof forms, proof by contradiction and mathematical induction.
  - solve discrete mathematics problems that involve: computing permutations and combinations of a set, fundamental enumeration principles.
  - evaluate Boolean functions and simplify expressions using the properties of Boolean algebra.
-

18UPMAT1E02	ANALYTIC NUMBER THEORY	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

The aim of this course is to teach the students about the basics of elementary number theory starting with the fundamental theorem of arithmetic, arithmetic functions, multiplicative functions, some equivalent forms of prime number theorem.

**UNIT I: The fundamental Theorem of Arithmetic**

Divisibility – greatest common divisor – prime numbers – the fundamental theorem of arithmetic – the series of reciprocals of the primes – the euclidean algorithm – the gcd of more than two numbers.

**UNIT II: Arithmetic functions and Dirichlet Multiplication**

The Möbius function  $\mu(n)$  – the Euler totient function  $\varphi(n)$  – a relation connecting  $\varphi$  and  $\mu$  - a product formula for  $\varphi(n)$  – the Dirichlet product of arithmetical functions – Dirichlet inverse and the Möbius inversion formula – the Mangoldt function  $\Lambda(n)$ .

**UNIT III: Multiplicative functions**

Multiplicative functions – multiplicative functions and Dirichlet multiplication – the inverse of a completely multiplicative function – Liouville’s function – the divisor functions – generalized convolutions.

**UNIT IV: Averages of Arithmetical Functions**

Asymptotic equality of functions – Euler’s summation formula – some elementary asymptotic formula – the average order of  $d(n)$  – average order of the divisor functions the average order of  $\varphi(n)$  – the average order of  $\mu(n)$  and of  $\Lambda(n)$ .

**UNIT V: Distribution of Prime Numbers**

The partial sums of a Dirichlet product – applications to  $\mu(n)$  and  $\Lambda(n)$  - Chebyshev’s functions  $\psi(x)$  and  $\mathcal{J}(x)$  – relations connecting  $\mathcal{J}(x)$  and  $\pi(x)$ . Some equivalent forms of the prime number theorem, inequalities for  $\Lambda(n)$  and  $p_n$ .

**TEXT BOOK:**

**Tom M. Apostol**, “*Introduction to Analytic Number Theory*”, Springer, International Student Edition, 2013.

UNIT	Chapter	Sections
I	1	full
II	2	2.1 – 2.8
III	2	2.9 – 2.14
IV	3	3.1 – 3.9
V	3	3.10, 3.11
	4	4.1 – 4.5

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **R.G. Ayoub**, “*An Introduction to the Analytic Theory of Numbers*”, Mathematical Surveys, No.10, Providence, R.I, AMS Publications, 1963.
2. **K. Chandrasekharan**, “*Introduction to Analytic Number Theory*”, Springer Verlag, 1968.
3. **D.T. Newman**, “*Analytic Number Theory*” GTM Vol 177, Correeted Edition, Springer, 2000.
4. **Heng Huat Chan**, “*Analytic Number Theory for undergraduate*” World Scientific, 2009.
5. **William Duke** and **Yuri Tschinkel**, “*Analysis Number Theory: A Tribute to Gauss and Dirichlet, Clay Mathematics*” Proceeding Vol. 7, AMS Publication, Providence, RI, 2007.
6. **H. Iwaniec**, **E. Kowalski**, “*Analytic Number Theory*” AMS Colloquium Publications, Vol. 53, AMS, 2004.

**LEARNING OUTCOMES:** At the end of the course, students will be able

- to find a practical method for computing the  $\gcd(a,b)$  when the prime-power factorizations of  $a$  and  $b$  are known.
- to find the interrelationships between various arithmetical functions.
- to find some elementary identities involving  $\mu(n)$  and  $\Lambda(n)$  which will be used in studying the distribution of primes.

<b>18UPMAT1E03</b>	<b>DIFFERENCE EQUATIONS</b>			
	L	T	P	C
	3	1	0	4

**OBJECTIVE:**

Difference equations usually describe the evolution of certain phenomena over the course of time. The aim of studying this course is

- ❖ To introduce the difference calculus.
- ❖ To study linear difference equations and to know how to solve them.
- ❖ To know the stability theory for homogeneous linear system of difference equations.
- ❖ To study the asymptotic behavior of solutions of homogeneous linear difference equations.

**UNIT I: Difference Calculus**

Difference operator - Summation – Generating functions and approximate summation.

**UNIT II: Linear Difference Equations**

First order equations - General results for linear equations - Solving linear equations.

**UNIT III: Linear Difference Equations**

Equations with variable coefficients – The  $z$  -transform.

**UNIT IV: Stability Theory**

Initial value problems for linear systems – Stability of linear systems.

## UNIT V: Asymptotic Methods

Introduction – Asymptotic analysis of sums – Linear equations.

### TEXT BOOK:

**W.G. Kelley** and **A.C. Peterson**, “*Difference Equations*”, 2<sup>nd</sup> Edition, Academic Press, New York, 2001.

UNIT	Chapter	Sections
I	2	2.1 – 2.3
II	3	3.1 – 3.3
III	3	3.5, 3.7
IV	4	4.1, 4.2
V	5	5.1 – 5.3

### BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:

1. **R.P. Agarwal**, “*Difference Equations and Inequalities*”, 2<sup>nd</sup> Edition, Marcel Dekker, New York, 2000.
2. **S.N. Elaydi**, “*An Introduction to Difference Equations*”, 3<sup>rd</sup> Edition, Springer, India, 2008.
3. **R. E. Mickens**, “*Difference Equations*”, 3<sup>rd</sup> Edition, CRC Press, 2015.

### LEARNING OUTCOMES:

After the successful completion of the course, students will be able

- to know the fundamentals of difference calculus, like, the difference operator, the computation of sums, the concept of generating function and the important Euler summation formula.
- to solve linear difference equations using different methods, namely, annihilator method, z-transform method, etc.
- to find the stability results for the linear system using eigen value criteria.
- to find asymptotic analysis of sums, and asymptotic behavior of solutions to linear difference equations by the theorems of Poincare and Perron.

---

18UPMAT1E04	NUMERICAL ANALYSIS	L	T	P	C
		4	1	0	4

### OBJECTIVES:

The objectives of this course are

- ❖ to make the students familiarize with the ways of solving complicated mathematical problems numerically.
- ❖ To provide numerical methods for solving the non-linear equations, interpolation, differentiation, integration, ordinary and partial differential equations.
- ❖ Describing and understanding error analysis in numerical methods.

**Unit I: Solutions of Equations in One Variable**

Newton's Method and its Extensions – Error Analysis for Iterative Methods – interpolation and Polynomial Approximation - Interpolation and the Lagrange Polynomial – Hermite Interpolation – Cubic Spline Interpolation.

**Unit II: Numerical Differentiation and Integration**

Numerical Differentiation – Elements of Numerical Integration – Romberg Integration.

**Unit III: Initial Value Problems for Ordinary Differential Equations**

Elementary Theory of Initial Value Problems – Euler's Method – Taylor Method – Runge-Kutta Methods.

**Unit IV: Initial Value Problems for Ordinary Differential Equations (Continued)**

Multistep Methods – Higher-Order Equations and Systems of Differential Equations – Stability.

**Unit V: Numerical Solutions to Partial Differential Equations**

Elliptic Partial Differential Equations – Parabolic Partial Differential Equations - Hyperbolic Partial Differential Equations.

**TEXT BOOK:**

**R. L. Burden and J.D. Faires**, “*Numerical Analysis*”, 9<sup>th</sup> Edition, Thomson Learning, Inc., Stanford, Connecticut, 2011.

UNIT	Chapter(s)	Sections
I	2 & 3	2.3, 2.4, 3.1, 3.4, 3.5
II	4	4.1, 4.3, 4.5
III	5	5.1, 5.2, 5.4
IV	5	5.6, 5.9, 5.10
V	12	12.1 – 12.3
<b>Algorithms are not included in the syllabus</b>		

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **C.F. Gerald and P.O. Wheatley**, “*Applied Numerical Analysis*” Sixth Edition, Addison-Wesley, Reading, 1998.
2. **M.K. Jain**, “*Numerical Methods for Scientific and Engineering Computation*” New Age International, 2003.

**LEARNING OUTCOMES:** At the end of the course, students will

- learn the principles for designing numerical schemes for differential equations.
- be able to analyze the consistency, stability and convergence of a numerical scheme.
- be able to know, for each type of differential equations, what kind of numerical methods are best suited for and the reasons behind these choices?
- be able to make a connection between the mathematical equations or properties and the corresponding physical meanings.
- be able to use a programming language or mathematical software to implement and test the numerical schemes.



18UPMAT1E05	METHODS OF APPLIED MATHEMATICS	L	T	P	C
		4	1	0	4

**OBJECTIVES:**

This course treats the foundations of calculus of variations and gives example on some applications within physics and engineering science: the Euler-Lagrange equation, the brachistochrone problem, minimal surfaces of revolution, Fermat's principle, Hamilton principle, Lagrange's and Hamilton's equations of motion, the Euler-Lagrange equation for several independent variables, vibrating strings and membranes, Ritz Optimisation, relation between differential and integral equations, the Green functions, Fredholm integral equations with separable kernels, classical Fredholm theory, the Neumann Series and resolvent kernels.

**UNIT I: Calculus of variations**

Maxima and Minima – The simplest case – Examples - Natural and transition boundary conditions – The variational notation – The more general case – Constraints and Lagranges multipliers – Variable end points – Sturm-Liouville problems.

**UNIT II: Applications of Calculus of variations**

Hamilton's principle – Lagrange's equation – Generalized dynamical entities – Constraints in dynamical systems – Small vibrations about equilibrium – Variational problems for deformable bodies – Rayleigh – Ritz method.

**UNIT III: Integral Equations**

Integral equations – Relations between differential and integral equations – The Green's function – Fredholm equations with separable kernels – Example.

**UNIT IV: Integral Equations**

Hilbert – Schmidt theory – Iterative method for solving equations of the second kind – The Neumann Series – Fredholm theory – Singular integral equations.

**UNIT V: Special devices**

Special devices – Iterative approximation to characteristic functions – Approximation of Fredholm equations by sets of algebraic equations.

**TEXT BOOK:**

**F.B. Hildebrand**, "Methods of Applied Mathematics", Prentice-Hall of India Pvt., New Delhi, 1968.

UNIT	Chapter	Sections
I	2	2.1 – 2.9
II	2	2.10 – 2.14, 2.16, 2.19
III	3	3.1 – 3 .3, 3.6, 3.7
IV	3	3.8 – 3.12
V	3	3.13 – 3.15

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **R.P. Kanwal**, “*Linear integral equation: Theory and Techniques*”, 2<sup>nd</sup> Edition, Birkhäuser, 1996.
2. **A.S. Gupta**, “*Calculus of Variations with Application*”, Prentice-Hall of India, New Delhi, 2005.
3. **L. Elsgolts**, “*Differential Equations and Calculus of Variations*”, University Press of the Pacific, 2003.

**LEARNING OUTCOMES:** At the end of the course, students will be able to

- give an account of the foundations of calculus of variations and of its applications in mathematics and physics.
- describe the brachistochrone problem mathematically and solve it.
- solve isoperimetric problems of standard type.
- solve simple initial and boundary value problems by using several variable calculus.
- use the theory, methods and techniques of the course to solve problems.

<b>18UPMAT1E06</b>	<b>OPTIMIZATION TECHNIQUES</b>	L	T	P	C
		4	1	0	4

**OBJECTIVES:**

- ❖ To introduce the methods of optimization techniques.
- ❖ To understand the theory of optimization techniques for solving various types of optimization problems.
- ❖ To provide with basic skills and knowledge of optimization techniques and their applications.
- ❖ To make the students familiar in solving techniques, analysing the results and propose recommendations to the decision-making processes.

**UNIT I: Linear Programming Problems**

Dual Simplex – Revised Simplex - Illustrative Applications - Integer Programming Algorithms.

**UNIT II: Decision Analysis and Games**

Decision Making under certainty – Decision Making under Risk – Decision under uncertainty – Game Theory.

**UNIT III: Inventory Models - Deterministic Models**

Inventory Models - Probabilistic Models.

**UNIT IV: Queuing Theory**

Elements of a Queuing model – Role of Exponential Distribution – Pure Birth and Death Models – Generalized Poisson Queuing Model – Specialized Poisson Queues – (M/G/1): (GD/∞/∞) – Pollaczek - Khintchine (P-K) Formula.

**UNIT V: Optimization Theory**

Classical Optimization Theory – Unconstrained Problems – Constrained Problems.

**TEXT BOOK:**

**Hamdy A Taha, Operations Research: An Introduction**, 7<sup>th</sup> Edition, Prentice – Hall of India, New Delhi, 2003.

UNIT	Chapter(s)	Sections
I	4 & 7	4.4, 7.2, 9.1, 9.2
II	14	14.1 – 14. 4
III	11 & 16	11.1 – 11.3, 16.1
IV	17	17.2 – 17.7 (Omit 17.6.4)
V	20	20.1, 20.2

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **F.S.Hillier and G.J.Lieberman**, *Introduction to Operations Research*, 4<sup>th</sup> Edition, Mc Graw Hill Book Company, New York, 1989.
2. **D.T. Philips, A. Ravindra and J. Solberg**, *Operations Research, Principles and Practice*, John Wiley and Sons, New York, 1991.
3. **B.E.Gillett**, *Operations Research – A Computer Oriented Algorithmic Approach*, TMH Edition, New Delhi, 1976.

**LEARNING OUTCOMES:** At the end of the course, students will be able to

- Formulate a real-world problem as linear programming and queuing models.
- Assess the existence and uniqueness of solutions and derive necessary and sufficient optimality conditions for a given optimization problem.
- Understand the mathematical tools that are needed to solve optimization problems.
- Identify and develop decision making and inventory models from the verbal description of the real system.

–

18UPMAT1E07	COMBINATORIAL MATHEMATICS	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

Combinatorial mathematics is the study of the arrangements of objects, according to prescribed rules, to count the number of possible arrangements or patterns, to determine whether a pattern of a specified kind exists and to find methods of constructing arrangements of a given type. The objective of this course is to acquaint the students with the concepts of permutations and combinatorics, generating functions, recurrence relations, the principle of inclusion and exclusion and Polya's theory of counting.

**UNIT I: Permutations and Combinatorics**

The Rules of sum and product – Permutations – Combinations – Distributions of distinct objects – Distribution of nondistinct objects – Stirling's formula

## UNIT II: Generating Functions

Generating functions for combinations – Enumerators for permutations- Distributions of distinct objects into nondistinct cells – Partitions of integers – The Ferrers graph – Elementary relations.

## UNIT III: Recurrence relations

Linear recurrence relations with constant coefficients – Solution by the technique of generating functions – A special class of nonlinear difference equations – Recurrence relations with two indices.

## UNIT IV: The Principle of inclusion and exclusion

The Principle of inclusion and exclusion – The general formula – Derangements – Permutations with restrictions on relative positions – The rook polynomials – Permutations with forbidden positions.

## UNIT V: Polya's theory of counting

Sets, relations and groups – Equivalence classes under a permutation group – Equivalence classes of functions – Polya's fundamental theorem – Generalization of Polya's theorem.

## TEXT BOOK

**C.L.Liu**, *“Introduction to Combinatorial Mathematics”*, McGraw Hill Book Company, New York, 1968.

UNIT	Chapter(s)	Sections
I	1	1.1 – 1.7
II	2	2.1 – 2.7
III	3	3.1 – 3.5
IV	4	4.1 – 4.7
V	5	5.1 – 5.7

## BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:

1. **Murray Edelberg and C. L. Liu**, *“Solutions to Problems in Introduction to Combinatorial Mathematics”*, MC Grow-Hill Book & Co., New York, 1968.
2. **R.P. Stanley**, *“Enumerative Combinatorics”*, Volume I, 2<sup>nd</sup> Edition, Cambridge Studies in Advanced Mathematics (Book 49)s, Cambridge University Press, 1997.
3. **P.J. Cameron**, *“Combinatorics: Topics, Techniques, Algorithms”*, Cambridge University Press, Cambridge, 1998.
4. **Miklos Bona**, *“A Walk through Combinatorics”*, World Scientific Publishing Company, 2002.
5. **M. Aigner**, *“A Course in Enumeration”*, Springer-Verlag, Heidelberg, 2007.
6. **J.H. Van Lint and R.M. Wilson**, *“A Course in Combinatorics”*, 2<sup>nd</sup> Edition, Cambridge University Press, Cambridge, 2001.

7.

**LEARNING OUTCOMES:** After completing the course, students will be able to

- Use formulas for counting basic combinatorial outcomes to construct solutions to complete combinatorial enumeration problems:
  - permutation, with and without repetitions;
  - combinations, with and without repetitions;
- Apply counting strategies to solve discrete probability problems.
- Use specialized techniques to solve combinatorial enumeration problems:
  - generating functions;
  - recurrence relations;
  - Inclusion-exclusion principle.

---

18UPMAT1E08	FUZZY SETS AND THEIR APPLICATIONS	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

The objective of this course is to introduce the basic ideas of Fuzzy Sets, Fuzzy sets versus crisp sets, operation on Fuzzy sets, Fuzzy arithmetic and methods of contracting fuzzy sets.

**UNIT I: Fuzzy sets**

Fuzzy sets – Basic types – basic concepts – Characteristics- Significance of the paradigm shift - Additional properties of  $\alpha$ -cuts.

**UNIT II: Fuzzy sets versus crisp sets**

Representation of Fuzzy sets- Extension principle of Fuzzy sets – Operation on Fuzzy Sets – Types of operation – Fuzzy complements.

**UNIT III: Operations on Fuzzy sets**

Fuzzy intersection – t-norms, Fuzzy unions – t conorms-Combinations of operations – Aggregation operations.

**UNIT IV: Fuzzy Arithmetic**

Fuzzy numbers – Linguistic variables – Arithmetic operation on intervals – Lattice of Fuzzy numbers.

**UNIT V: Constructing Fuzzy Sets**

Methods of construction: an overview – direct methods with one expert – direct method with multiple experts – indirect method with multiple experts and one expert- Construction from sample data.

**TEXT BOOK:**

**G.J. Klir and Bo Yuan**, “*Fuzzy Sets and Fuzzy Logic: Theory and Applications*”, Prentice Hall of India Ltd, New Delhi, 2005.

UNIT	Chapter(s)	Sections
I	1 & 2	1.3 – 1.5, 2.1
II	2 & 3	2.2, 2.3, 3.1, 3.2
III	3	3.3 – 3.6
IV	4	4.1 – 4.4
V	10	10.1 – 10.7

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **H.J.Zimmermann**, “*Fuzzy Set Theory and its Applications*”, Allied Publishers, Chennai, 1996.
2. **A.Kaufman**, “*Introduction to the Theory of Fuzzy Subsets*”, Academi Press, New York, 1975.
3. **V.Novak**, “*Fuzzy Sets and Their Applications*”, Adam Hilger, Bristol, 1969.

**LEARNING OUTCOMES:** At the end of the course, students will be

- able to distinguish between the crisp set and fuzzy set concepts.
- able to draw a parallelism between crisp set operations and fuzzy set operations through the use of characteristic and membership functions, respectively.
- able to define fuzzy sets using linguistic words and represent these sets by membership functions.
- able to know how to perform mapping of fuzzy sets by a function and also use the  $\alpha$ -level sets in such instances.
- able to know the concept of a fuzzy number and how it is defined.
- become aware of the use of fuzzy inference systems in the design of intelligent or humanistic systems.
- aware of the applications of fuzzy inference in the area of control.

<b>18UPMAT1E09</b>	<b>REPRESENTATION THEORY OF FINITE GROUPS</b>	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

Representation theory, the art of realizing a group in a concrete way, usually as a collection of matrices, is a fundamental tool for studying groups by means of linear algebra. The results of the theory of representations of finite groups play a fundamental role in many recent developments of mathematics and theoretical physics. The study of the representation theory of groups becomes a special case of the study of modules over rings. This course provides the concepts of the characters of groups and the basic properties of irreducible characters and their connection with the ring structure of group algebras.

### UNIT I: Group representations

Group representations – FG Modules – FG - Submodules and Reducibility- Group algebras.

### UNIT II: Group algebra

FG-homomorphisms – Maschke’s theorem – Consequences of Maschke’s theorem – Schur’s lemma – Irreducible modules and the group algebra.

### UNIT III: More on the group algebra

More on the group algebra – The spaces of FG-homomorphisms – Conjugacy classes - Conjugacy class sizes – Characters – The values of a character – The regular character.

### UNIT IV: Irreducible characters

Inner product of characters – Applications – Decomposing CG-modules – Class functions – The number of irreducible characters.

### UNIT V: Character tables

Character Tables and Orthogonality relations- Normal subgroups and Lifted characters- Some Elementary Character Tables.

### TEXT BOOK:

**G.James** and **M.Liebeck**, “*Representations and Characters of Groups*”, 2<sup>nd</sup> Edition, Cambridge University Press, London, 2001.

UNIT	Chapter(s)
I	3 – 6
II	7 – 10
III	11 – 13
IV	14 – 15
V	16 – 18

### BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:

1. **C.W. Curtis** and **I.Reiner**, “*Methods of Representation Theory with Applications to Finite Groups and Orders*”, Volume 1, Wiley – Interscience, New York, 1981.
2. **J.P. Serre**, “*Linear Representation of Finite Groups*”, Springer-Verlag, New York, 1977.
3. **W.Fulton** and **J. Harris**, “*Representation Theory – A First Course*”, Graduate Texts in Mathematics 129, Springer – Verlag, New York, 1991.

**LEARNING OUTCOMES:** After completion of their course students will be

- able to see the special role played by the famous Maschke’s theorem.
- able to know the consequence of the Maschke’s theorem, the representation theory of groups splits into two different cases depending on the characteristic of a field  $k$ : classical and modular.

<b>18UPMAT1E10</b>	<b>NON COMMUTATIVE ALGEBRA – I</b>	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

The objective of this course is to equip the students with knowledge of some advanced concepts namely decomposition of rings, Artinian rings, Noetherian rings, categories, functors, projective, injective and flat modules and homological dimensions. This course also provides the foundation required for more advanced study in Algebra.

**UNIT I: Decompositions of Rings:**

Modules and homomorphisms – Classical isomorphism theorems – direct sums and products – free modules – two sided Peirce decomposition of a ring – the Wedderburn – Artin theorem – finitely decomposable rings.

**UNIT II: Artinian and Noetherian Rings:**

The Jordan-Holder theorem – the Hilbert basis theorem – the radical of a module and a ring – the radical of an Artinian rings – Semiprimary rings.

**UNIT III: Categories and Functors:**

Exact sequences – direct sums and direct products – the Hom functors – tensor product functor – direct and inverse limits.

**UNIT IV: Projectives, Injectives and Flats:**

Projective modules – injective modules – essential extensions and injective hulls – flat modules – right hereditary and right semihereditary rings – Herstein-Small rings.

**UNIT V: Homological Dimensions:**

Complexes and homology, free solutions – Projective and Injective resolutions, Derived functors – the functors Tor, EXT<sub>n</sub>, projective and injective dimensions – global dimensions.

**TEXT BOOK:**

**M. Hazewinkel, N. Gubareni and V.V. Kirichenko**, “Algebras, Rings and Modules”, Volume I, Springer International Edition, New Delhi, 2011.

UNIT	Chapter(s)	Sections
I	1 & 2	1.1 – 1.5, 2.1 – 2.4
II	3	3.1 – 3.7
III	4	4.1 – 4.7
IV	5	5.1 – 5.6
V	6	6.1 – 6.6



**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **T.Y. Lam**, “*Lectures on Modules and Rings*”, Graduate Texts in Mathematics, Vol. 189, Springer-Verlag, Berlin-Heidelberg, New York, 1999.
2. **J. Lambek**, “*Lectures on Rings and Modules*”, 3<sup>rd</sup> Edition, AMS Chelsea Publishing, AMS, Providence, Rhode Island, 2009.
3. **D.S. Passman**, “*A Course in Ring Theory*”, AMS Chelsea Publishing, AMS, Providence, Rhode Island, 2004.
4. **L.R. Veramani**, “*An Elementary Approach to Homological Algebra*”, Chapman & Hall / CRC Monographs and Surveys in Pure and Applied Mathematics. Vol. 130, CRS Press, LLC, Florida, 2003.

**LEARNING OUTCOMES:** At the end of course, students will be

- able to find whether the given module is injective or not? by using many structure theorems for injective modules.
- able to find the projective, injective and flat dimensions of modules.

<b>18UPMAT1E11</b>	<b>NONCOMMUTATIVE ALGEBRA – II</b>	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

The objective of this course is to equip the students with knowledge of some advanced concepts namely Dedekind domains, Goldie rings, Semi perfect rings and serial rings. This course also provides the foundation required for more advanced study in Algebra.

**UNIT I: Integral Domains**

Principal ideal domains – factorial rings – Euclidean domains – Rings of fractions and quotient fields – Polynomial rings over fractional rings – Gauss Lemma – Chinese remainder theorem – Smith normal form over a PID – Finitely generated modules over a PID – The Frobenius theorem.

**UNIT II: Dedekind domains**

Integral closure – Dedekind domains – Hereditary domains – Discrete valuation rings – Finitely generated modules over Dedekind domains – Purifier rings.

**UNIT III: Goldie rings**

The Ore condition – Classical rings of fractions – Prime and semiprime rings – Goldie rings – Goldie’s theorem.

**UNIT IV: Semiperfect rings**

Local and semilocal rings – noncommutative discrete valuation rings – Lifting idempotent – semiperfect rings – Projective covers – the Krull-Schmidt theorem – Perfect rings.

**UNIT V: Serial rings**

Equivalent Categories – Progenerator – The Morita theorem – Finitely presented modules – The Drozd-Warfield theorem – The Ore condition for serial rings – minors of serial right Noetherian rings.

**TEXT BOOK:**

**M. Hazewinkel, N. Gubareni and V.V. Kirichenko**, “*Algebras, Rings and Modules*”, Volume I, Springer International Edition, New Delhi, 2011.

UNIT	Chapter(s)	Sections
I	7	
II	8	
III	9	
IV	10	10.1 – 10.5
V	10 & 13	10.6, 10.7, 13.1 – 13.3
<b>Proposition 13.3.5 is omitted</b>		

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **P.E. Bland**, “*Rings and their Modules*”, Walter de GmbH Co.KG, Berlin New York, 2011.
2. **T.Y Lam**, “*Lectures on Rings and Modules*”, Springer-Verlag, New York, Berlin, 1998.
3. **J.C. Mc Connel and J.C. Robson**, “*Noncommutative Noetherian Rings*”, Graduate Studies in Mathematics, Vol.30, AMS, Providence, 2001.
4. **D. Passman**, “*A Course in Ring Theory*”, AMS, Providence, 2004.
5. **G. Puninski**, “*Serial Rings*”, Kluwer Academic Publishers, Dordecht, 2001.

**LEARNING OUTCOMES:** At the end of the course, students will be

- able to know the decomposition theorem which gives the structure of Noetherian Serial rings.
- able to find the full description of semiperfect two-sided Noetherian and hereditary prime rings.

<b>18UPMAT1E12</b>	<b>COMMUTATIVE ALGEBRA</b>	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

The objective of this course is to study modules, exact sequences, tensor product of modules, local properties, primary decomposition, Noetherian rings and Artinian rings. Also, another important class of Dedekind domain is studied.

**Unit I: Rings and Ideals**

Rings and ring homomorphism's – ideals – Extension and Contraction, modules and module homomorphism – exact sequences.

## Unit II: Rings and Modules of Fractions

Tensor product of modules – Tensor product of algebra – Local properties – extended and contracted ideals in rings of fractions.

## Unit III: Primary Decomposition

Primary Decomposition – Integral dependence – The going-up theorem – The going-down theorem – Valuation rings.

## Unit IV: Noetherian rings

Chain conditions – Primary decomposition in Noetherian rings.

## Unit V: Artin local rings

Artin rings – Discrete valuation rings – Dedekind domains – Fractional ideals.

### TEXT BOOK:

**S.M.Atiyah** and **I.G.Macdonald**, “*Introduction to Commutative Algebra*”, Addison – Wesley Publication Company, Inc, 1969.

UNIT	Chapter(s)	Pages
I	1, 2	1 - 24
II	2, 3	24 - 49
III	4, 5	50 - 73
IV	6, 7	74 - 88
V	8, 9	89 - 99

### BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:

1. **N.S. Gopalakrishnan**, “*Commutative Algebra*”, Oxonian Press Pvt. Ltd, New Delhi, 2015.
2. **I. Kaplansky**, “*Commutative Rings*”, University of Chicago Press, Chicago, 1974.
3. **H. Matsumura**, “*Commutative Ring Theory*”, Cambridge University Press, 1986.

**LEARNING OUTCOMES:** After completing the course, students will be able

- to know the definition of commutative rings, local rings, prime and maximal ideals and modules over commutative rings.
- to know the notions of Noetherian and artinian rings and modules.
- to know how to localize rings and modules, and the important applications of localization.
- to know the Hilbert basic theorem.
- to know important properties and applications of exact sequences.
- to know how to define tensor products of modules and the concept of flatness.

<b>16UPMAT1E13</b>	<b>CONTROL THEORY</b>	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

This is an introductory course in mathematical systems theory. The subject provides the mathematical foundation of modern control theory. The aim of the course is to acquire a systematic understanding of linear dynamical systems. The acquirement of such knowledge is useful in preparation for work on system analysis and design problems that appear in many engineering fields.

**Unit-I: Observability**

Linear Systems – Nonlinear Systems.

**Unit-II: Controllability**

Linear systems – Nonlinear systems.

**Unit-III: Stability**

Stability – Perturbed linear systems – Nonlinear systems.

**Unit IV: Stabilizability**

Stabilization via linear feedback control – The controllable subspace – Stabilization with restricted feedback.

**Unit V: Optimal Control**

Linear time varying systems – Linear time invariant systems – Nonlinear Systems.

**TEXT BOOK**

**K.Balachandran** and **J.P.Dauer**, “*Elements of Control Theory*”, 2<sup>nd</sup> Edition (revised), Alpha Science International Ltd, 2011.

<b>UNIT</b>	<b>Chapter(s)</b>	<b>Sections</b>
<b>I</b>	<b>2</b>	<b>2.1 – 2.3</b>
<b>II</b>	<b>3</b>	<b>3.1, 3.2</b>
<b>III</b>	<b>4</b>	<b>4.1 – 4.3, 4.5</b>
<b>IV</b>	<b>5</b>	<b>5.1 – 5.4</b>
<b>V</b>	<b>6</b>	<b>6.1 – 6.3</b>

**Books for Supplementary Reading and REFERENCES:**

1. **R. Conti**, “*Linear Differential Equations and Control*”, Academic Press, London, 1976.
2. **R.F. Curtain** and **A.J.Pritchard**, “*Functional Analysis and Modern Applied Mathematics*”, Academic Press, New York, 1977.
3. **J. Klamka**, “*Controllability of Dynamical Systems*”, Kluwer Academic Publisher, Dordrecht, 1991.

**LEARNING OUTCOMES:** At the end of the course, students

- will learn some basic notions and results in control theory, which are very useful for applied mathematicians.
- will get an understanding of the building blocks of basic and modern control systems.
- will get an understanding of the basic ingredients of linear systems theory and how these are used in analysis and design of control, estimation and filtering systems.
- will be able to select appropriate methodologies for the analysis or design of feedback and open-loop control systems.

---

<b>16UPMAT1E14</b>	<b>STOCHASTIC DIFFERENTIAL EQUATIONS</b>	L	T	P	C
		4	1	0	4

**Prerequisites:** Probability theory, Differential equations and Real analysis

**OBJECTIVE:**

Stochastic differential equations have been used extensively in many areas of application, including finance and social science as well as in physics, chemistry. This course develops the theory of Itô's calculus and stochastic differential equations.

**Unit I: Mathematical Preliminaries and Itô Integrals**

Probability Spaces – Random variables and Stochastic Processes – An Important Example: Brownian motion – Construction of the Itô Integral – Some Properties of the Itô Integral – Extensions of the Itô Integral.

**Unit II: Itô Formula and Martingale Representation Theorem**

The 1-dimensional Itô Formula - The Multi-dimensional Itô Formula – The Martingale Representation Theorem.

**Unit III: Stochastic Differential Equations**

Examples and Some Solution Methods – An Existence and Uniqueness Result – Weak and Strong Solutions.

**Unit IV: The Filtering Problem**

Introduction – The 1-Dimensional Linear Filtering Problem – The Multidimensional Linear Filtering Problem.

## Unit V: Diffusions: Basic Properties

The Markov Property – The Strong Markov Property – The Generator of an Itô Diffusion – The Dynkin Formula – The Characteristic Operator.

### TEXT BOOK:

**B. Oksendal**, “*Stochastic Differential Equations: An Introduction with Applications*”, 6<sup>th</sup> Edition, Springer - Verlag, Heidelberg, 2003.

UNIT	Chapter(s)	Pages
I	2 & 3	2.1, 2.2, 3.1 – 3.3
II	4	4.1 – 4.3
III	5	5.1 – 5.3
IV	6	6.1 – 6.3
V	7	7.1 – 7.5

### BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:

1. **A. Friedman**, “*Stochastic Differential Equations and Applications*”, Dover Publications, 2006.
2. **L. Arnold**, “*Stochastic Differential Equations: Theory and Applications*”, Dover Publications, 2011.
3. **D. Henderson** and **P. Plachko**, “*Stochastic Differential Equations in Science and Engineering*”, World Scientific, 2006.

**LEARNING OUTCOMES:** At the end of the course, students

- will have a thorough understanding of stochastic methods that are in between mathematical analysis and probability theory.
- will get knowledge of the Ito stochastic integral and the related stochastic differential equations that will help to pursue research in this area.

18UPMAT1E15	NUMBER THEORY	L	T	P	C
		4	1	0	4

### OBJECTIVE:

The aim of this course is to teach the students about the basics of elementary number theory starting with primes, congruences, quadratic residues, primitive roots, arithmetic functions and some Diophantine equations.

### Unit I: Divisibility and Congruences

Divisibility – Primes – Congruences – Solutions of Congruences.

### Unit II: Congruences

The Chinese Remainder Theorem – Prime Power Moduli – Prime Modulus - Primitive Roots and Power Residues – Congruences of Degree Two, Prime Modulus.

### Unit III: Quadratic Reciprocity and Quadratic Forms

Quadratic Residues – Quadratic Reciprocity – The Jacobi Symbol – Sums of Two Squares.

### Unit IV: Some Functions of Number Theory

Greatest Integer Function – Arithmetic Functions – The Mobius Inversion Formula – Combinatorial Number Theory.

### Unit V: Some Diophantine Equations

The Equation  $ax + by = c$  – Simultaneous Linear Equations – Pythagorean Triangles – Assorted Examples.

#### TEXT BOOK:

**I. Niven, H. S. Zuckerman and H. L. Montgomery, An Introduction to the Theory of Numbers**, 5<sup>th</sup> Edition, John Wiley & Sons, Inc., New York, 2004.

UNIT	Chapter(s)	Sections
I	1 & 2	1.2, 1.3, 2.1, 2.2
II	2	2.3, 2.6 – 2.9
III	3	3.1 – 3.3, 3.6
IV	4	4.1 – 4.3, 4.5
V	5	5.1 – 5.4

#### BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:

1. **D.M. Burton**, *Elementary Number Theory*, Universal Book, Stall, New Delhi 2001.
2. **K. Ireland and M. Rosen**, *A Classical Introduction to Modern Number Theory*, Springer Verlag, New York, 1972.
3. **T.M. Apostol**, *Introduction to Analytic Number Theory*, Narosa Publ. House, Chennai, 1980.

**LEARNING OUTCOMES:** At the end of the course, student will be able to

- apply the Law of Quadratic Reciprocity and other methods to classify numbers as primitive roots, quadratic residues and quadratic non-residues.
- formulate and prove conjectures about numeric patterns and
- produce rigorous arguments centered on the material of number theory, most notably in the use of Mathematical induction and the Well-Ordinary principle in the proof of theorems.

18UPMAT1E16	DIFFERENTIAL GEOMETRY	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

This course gives students basic knowledge of classical differential geometry of curves and surfaces such as the catenary, the tractrix, the cycloid and the surfaces of constant Gaussian curvature and minimal surfaces. .

**UNIT I: Space Curves**

Definition of a space curve – Arc length – Tangent – Normal and binormal – Curvature and torsion – Contact between curves and surfaces – Tangent surface – Involutives and evolutes – Intrinsic equations – Fundamental existence theorem for space curves – Helices.

**UNIT II: Intrinsic Properties of a Surface**

Definition of a surface – Curves on a surface – Surface of revolution – Helicoids – Metric – Direction coefficients – Families of curves – Isometric correspondence – Intrinsic properties.

**UNIT III: Geodesics**

Geodesics – Canonical geodesic equations – Normal property of geodesics – Existence theorems – Geodesic parallels – Geodesics curvature- Gauss-Bonnet Theorem – Gaussian curvature – Surface of constant curvature.

**UNIT IV: Non Intrinsic Properties of a Surface**

The second fundamental form – Principal curvature – Lines of curvature – Developable – Developable associated with space curves and with curves on surface – Minimal surfaces – Ruled surfaces.

**UNIT V: Differential Geometry of Surfaces**

Compact surfaces whose points are umbilics – Hilbert’s lemma – Compact surface of constant curvature – Complete surface and their Characterization – Hilbert’s Theorem – Conjugate points on geodesics.

**TEXT BOOK:**

**T.J. Willmore**, “An Introduction to Differential Geometry”, Oxford University press, (17<sup>th</sup> Impression), New Delhi, 2002. (Indian Print)

UNIT	Chapter(s)	Sections
I	I	1 – 9
II	II	1 – 9
III	II	10 – 18
IV	III	1 – 8
V	IV	1 – 8



**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **D.T. Struik**, “*Lectures on Classical Differential Geometry*”, Addison –Wesley, Mass, 1950.
2. **S. Kobayashi** and **K. Nomizu**, “*Foundations of Differential Geometry*”, Interscience Publishers, 1963.
3. **W. Klingenberg**, “*A Course in Differential Geometry*”, Graduate Texts in Mathematics, Springer – Verlag 1979.
4. **C.E. Weatherburn**, “*Differential Geometry of Three Dimensions*”, University Press, Cambridge, 1930.

**LEARNING OUTCOMES:** After successful completion of the course, students will be able to

- calculate the curvature and torsion of a curve.
- find the osculating surface and osculating curve at any point of a given curve.
- calculate the first and the second fundamental forms of surface.
- calculate the Gaussian curvature, the mean curvature, the curvature lines, the asymptotic lines, the geodesics of a surface.

<b>18UPMAT1E17</b>	<b>ADVANCED PARTIAL DIFFERENTIAL EQUATIONS</b>	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

The objective is to

- ❖ develop an understanding of the theory and methods of solution for partial differential equations.
- ❖ provide an introduction to the study and solution methods for partial differential equations of first and second order.
- ❖ make the students to understand the characteristics of heat, wave, and Laplace’s equations.
- ❖ provide the students a better understanding to the diffusion and wave equations and their applications.

**Unit-I: Laplace Equation**

Partial Differential Equations – Classifications – Examples - Fundamental solution – Mean-value formulas – Properties of harmonic functions – Green’s functions – Energy methods.

**UNIT II: Heat Equation**

Fundamental solution – Mean-value formula – Properties of solutions – Energy methods.

**UNIT III: Wave Equation**

Solution by spherical means – Nonhomogeneous problem – Energy methods.

**UNIT IV: Other ways to represent solutions**

Separation of variables - Similarity solutions.

## UNIT V: Other ways to represent solutions

Transform methods - Converting nonlinear into linear PDE.

### TEXTBOOK:

**L. C. EVANS**, “*Partial Differential Equations*”, American Mathematical Society, Indian Edition, 2009.

UNIT	Chapter(s)	Sections
I	1 & 2	1.1, 1.2, 2.2
II	2	2.3
III	2	2.4
IV	4	4.1, 4.2
V	4	4.3, 4.4

### BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:

1. **D. Colton**, “*Partial Differential Equations: An Introduction*”, Dover Publishers, New York, 1988.
2. **F. John**, “*Partial Differential Equations*”, Applied Mathematical Science (Vol. 1), Springer, 1982.
3. **M. Renardy** and **R.C.Rogers**, “*An Introduction to Partial Differential Equations*”, Springer, 2004.
4. **R. McOwen**, “*Partial Differential Equations: Methods and Applications*”, 2<sup>nd</sup> Edition, Pearson Education, 2005.

**LEARNING OUTCOMES:** After the successful completion of the course, students will be able to

- enhance their mathematical understanding in representing solutions of partial differential equations.
- understand the fundamental theory of partial differential equations to enter in to research career.

18UPMAT1E18	NONLINEAR DIFFERENTIAL EQUATIONS	L	T	P	C
		4	1	0	4

**OBJECTIVE:** The main objective of this course is

- ❖ to discuss nonlinear ordinary differential equations for their different behavior of the solutions.
- ❖ to study periodic solutions and averaging methods, perturbation methods and stability.
- ❖ to analyze some applications of nonlinear ordinary differential equations studied in the present work to some concrete problem of the other areas of mathematics.

## UNIT I: Plane autonomous systems and linearization

The general phase plane - Some population models – Linear approximation at equilibrium points – Linear systems in matrix form.

**UNIT II: Periodic Solutions and Averaging Methods**

An energy balance method for limit cycles – Amplitude and frequency estimates – Slowly varying amplitudes; Nearly periodic solutions - Periodic solutions: Harmonic balance – Equivalent linear equation by harmonic balance – Accuracy of a period estimate.

**UNIT III: Perturbation Methods**

Outline of the direct method – Forced oscillations far from resonance- Forced oscillations near resonance with weak excitation – Amplitude equation for undamped pendulum – Amplitude perturbation for the pendulum equation – Lindstedt’s method – Forced oscillation of a self – excited equation – The Perturbation method and Fourier series.

**UNIT IV: Stability**

Poincare stability – Paths and solution curves for general systems - Stability of time solutions: Liapunov stability - Liapunov stability of plane autonomous linear systems

**UNIT V: Stability**

Structure of the solutions of  $n$ -dimensional linear systems - Structure of  $n$ -dimensional inhomogeneous linear systems - Stability and boundedness for linear systems - Stability of linear systems with constant coefficients.

**TEXT BOOK:**

**D.W.Jordan** and **P.Smith**, “*Nonlinear Ordinary Differential Equations*”, 4<sup>th</sup> Edition, Oxford University Press, New York, 2007.

UNIT	Chapter	Sections
I	2	2.1 – 2.5
II	4	4.1 – 4.5
III	5	5.1 – 5.5, 5.8 – 5.11
IV	8	8.1 – 8.4
V	8	8.5 – 8.8

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **G.F. Simmons**, “*Differential Equations*”, Tata McGraw-Hill, New Delhi, 1995.
2. **D.A. Sanchez**, “*Ordinary Differential Equations and Stability Theory*”, Dover, New York, 1979.
3. **J.K. Aggarwal**, “*Notes on Nonlinear Systems*”, Van Nostrand, 1972.

**LEARNING OUTCOMES:** At the end of the course, students will

- understand and apply the concept of stability of a fixed point solution of a system of ordinary differential equations in various science aspects.

18UPMAT1E19	MATHEMATICAL BIOLOGY	L	T	P	C
		4	1	0	4

**OBJECTIVE:**

Biology is undergoing a quantitative revolution, generating vast quantities of data that are analysed using bioinformatics techniques and modelled using mathematics to give insight into the underlying biological processes. This module aims to give a flavour of how mathematical modelling can be used in different areas of biology.

**UNIT I: Single Species Population Dynamics**

Continuous time models – Growth models, Logistic model – Evolutionary Aspects – Delay models.

**UNIT II: Two Species Population Dynamics**

The Lotka-Volterra Prey-Predator equations – Modelling the predator functional response  
Competition – Ecosystems modeling.

**UNIT III: Infectious Diseases**

Simple epidemic and SIS diseases – SIR Epidemics – SIR Endemics.

**UNIT IV: Biochemical Kinetics**

Transitions between states at the molecular and populations level – Law of mass action – Enzyme kinetics.

**UNIT V: Biochemical Kinetics**

Simple models for polymer growth dynamics.

**TEXT BOOK:**

1. **N. Britton**, “*Essential Mathematical Biology*”, Springer Science & Business Media, 2012.
2. **L.A. Segel** and **L. Edelstein-Keshet**, “*A Primer in Mathematical Models in Biology*”, SIAM, Vol. 129, 2013.

UNIT	Chapter/ Text Book	Section(s)
I	1 of [1]	1.3 – 1.5, 1.7
II	2 of [1]	2.3 - 2.6
III	3 of [1]	3.1 - 3.4
IV	2 of [2]	2.1 - 2.4
V	2 of [2]	2.5

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **J.D. Murray**, “*Mathematical Biology I: An Introduction*”, Springer-Verlag, New York, 2002.
2. **A. D. Bazykin**, “*Nonlinear dynamics of interacting populations*”, World Scientific, 1998.
3. **J.N.Kapur**, “*Mathematical Models in Biology and Medicine*”, Affiliated East-West, New Delhi, 1985.

**LEARNING OUTCOMES:** At the end of the course, students will be able to

- solve mathematically and interpret biologically simple problems involving one- and two-species ecosystems, epidemics and biochemical reactions.

\_\_\_\_\_

<b>18UPMAT1E20</b>	<b>FLUID DYNAMICS</b>	L	T	P	C
		4	1	0	4

**OBJECTIVE:** The objective of this course is

- To give fundamental knowledge of fluid, its properties and behavior under various conditions of internal and external flows.
- To understand basic laws and equations used for analysis of static and dynamic fluids.
- To develop an appreciation for the properties of Newtonian fluids.
- To understand the dynamics of fluid flows and the governing non-dimensional parameters

**Unit I: Inviscid Theory**

Introductory Notions, velocity: Streamlines and paths of the particles-stream tubes and filaments-fluid body- Density – Pressure – Bernoulli’s theorem. Differentiation with respect to time- Equation of continuity- Boundary conditions: kinematical and physical – Rate of change of linear momentum – The equation of motion of an inviscid fluid.

**Unit II: Inviscid Theory (contd...)**

Euler’s momentum theorem- conservative forces – Lagrangian form of the equation of motion – Steady motion – The energy equation – Rate of change of circulation – Vortex motion – Permanence of vorticity.

**Unit III: Two Dimensional Motion**

Two dimensional functions: Stream function – Velocity potential – Complex potential – Indirect approach – Inverse function. Basic singularities: Source – Doublet – Vortex – Mixed flow – Method of images: Circle theorem – Flow past circular cylinder with circulation. The aerofoil: Blasius’s theorem – Lift force.

**Unit IV: Viscous Theory**

The equations of motion for viscous flow: The stress tensor – The Navier-Stokes equations – Vorticity and circulation in a viscous fluid. Flow between parallel flat plates: Couette flow, Plane Poiseuille flow. Steady flow in pipes: Hagen-Poiseuille flow.

## Unit V: Boundary Layer Theory

Boundary layer concept- Boundary layer equations in two dimensional flow- Boundary layer along a flat plate: Blasius solution – Shearing stress and boundary layer thickness – Momentum integral theorem for the boundary layer: The von Karman integral relation – von Karman integral relation by momentum law.

### TEXT BOOKS:

1. **L.M. Milne Thomson**, “*Theoretical Hydrodynamics*”, Dover, 1996.
2. **N. Curle and H.J. Davies**, “*Modern Fluid Dynamics Vol-I*” by, D Van Nostrand Company Ltd., London, 1968.
3. **S.W. Yuan**, “*Foundations of Fluid Mechanics*” by Prentice- Hall of India, New Delhi, 1988.

UNIT	Chapter(s)	Sections
I	I & III of [1]	1.0 – 1.4, 3.10 – 3.31, 3.40, 3.41
II	III of [1]	3.42 – 3.45, 3.50 – 3.53
III	3 of [2]	3.2, 3.3, 3.5 - 3.5.1, 3.5.2, 3.7.4, 3.7.5
IV	5 of [2]	5.2.1- 5.2.3
	8 of [3]	8.3 – a,b, 8.4 – a
V	9 of [3]	9.1, 9.2, 9.3 – a,b, 9.5 – a,b

### BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:

1. **R.K. Bansal**, “*An Introduction to Fluid Dynamics*”, Firewall Media, 2005.
2. **G.K. Batchelor**, “*An Introduction to Fluid Dynamics*”, Cambridge University Press, 2000.
3. **F. Chorlton**, “*Text Book of Fluid Dynamics*”, CBS Publications, Delhi, 1985.
4. **D.E. Rutherford**, “*Fluid Dynamics*”, Oliver and Boyd, 1959.

**LEARNING OUTCOMES:** On successful completion of the course, the student will be able to,

- Recognize and find the values of fluid properties and relationship between them and understand the principles of continuity, momentum, and energy as applied to fluid motions.
- Identify these principles written in form of mathematical equations.
- Apply dimensional analysis to predict physical parameters that influence the flow in fluid mechanics.

18UPMAT1S01	APPLIED MATHEMATICS – I	L	T	P	C
		3	0	0	3

**OBJECTIVE:**

The objective of this course is to provide a strong foundation for partial differential equation and obtaining its solutions using classical methods.

**UNIT I: Ordinary Differential Equations**

Second and higher order linear ODE – Homogeneous linear equations with constant and variable coefficients – Nonhomogeneous equations – Solutions by variation of parameters.

**UNIT II: Functions of Several Variables**

Partial derivatives – Total differential – Taylor’s expansions – Maxima and Minima of functions – Differentiation under integral sign.

**UNIT III: Partial Differential Equations**

Formation of PDE by elimination of arbitrary constants and functions – Solutions – General and singular solution- Lagrange’s Linear equation – Linear PDE of second and higher order with constant coefficients.

**UNIT IV: Fourier Series**

Dirichlet’s conditions – General Fourier series – Half range Sine and Cosine series –Parseval’s identity – Harmonic Analysis.

**UNIT V: Boundary Value Problems**

Classifications of PDE – Solutions by separation of variables - One dimensional heat and wave equation.

**TEXT BOOK:**

1. **B.S. Grewal**, “*Higher Engineering Mathematics*”, 30<sup>th</sup> Eighth Edition, Khanna Publishers, Delhi, 2004.
2. **E. Kreyszig**, “*Advanced Engineering Mathematics*”, 8<sup>th</sup> Edition, John Wiley and Sons, (Asia), Singapore, 2000.

**LEARNING OUTCOMES:**

- At the end of the course, students will be able to solve simple ordinary and partial differential equations.

18UPMAT1S02	APPLIED MATHEMATICS – II	L	T	P	C
		3	0	0	3

**OBJECTIVE:**

The objective of this course is to provide the strong background of applicable mathematics

**UNIT I: Laplace Transform**

Transform of elementary functions – Transforms of derivatives and integrals – Initial and final value theorems – Inverse Laplace transform – Convolution theorem – Solutions of linear ODE with constant coefficients.

**UNIT II: Fourier Transforms**

Fourier integral theorem – Fourier transform pairs– Fourier Sine and Cosine transforms – Properties – Transforms of simple functions – Convolution theorem – Parseval’s identity.

**UNIT III: Multiple Integrals**

Double integration – Cartesian and polar co-ordinates – Change of order of integration – Area as a double integral – Triple integration – Volume as a triple integral.

**UNIT IV: Vector Calculus**

Gradient, Divergence and Curl – Directional derivative – Irrotational and solenoid vector fields – Vector integration – Green’s theorem, Gauss divergence theorem and Stoke’s theorem.

**UNIT-V: Numerical Solutions of ODEs**

Solution by Taylor’s series method – Euler’s method – Modified Euler method, Runge-Kutta Method – Solving simultaneous equations.

**TEXT BOOK:**

1. **E. Kreyszig**, “*Advanced Engineering Mathematics*”, 8<sup>th</sup> Edition, John Wiley and Sons, Singapore, 2000.
2. **B.S. Grewal**, “*Higher Engineering Mathematics*”, 30<sup>th</sup> Edition, Khanna Publishers, Delhi 2004.

**LEARNING OUTCOMES:**

- At the end of the course, students will be able to understand the mathematical transformations, fundamental theorem of vector calculus, evaluate multiple integrals and to solve ordinary differential equations using numerical techniques.



18UPMAT1S03	NUMERICAL & STATISTICAL METHODS	L	T	P	C
		3	0	0	3

**OBJECTIVE:**

The objective of this course is to provide the foundation for numerical methods and statistics.

**UNIT I: Algebraic and Transcendental Equations**

Bisection Method – Iteration Method – The Method of False Position – Newton- Raphson – Method.

**UNIT II: System of Linear Equation**

Gauss Elimination, Gauss Jordan elimination – Triangularization method –Iterative Methods, Jacobi, Gauss-Seidal iteration, Iterative method for  $A^{-1}$ .

**UNIT III: Interpolation**

Interpolation with equal intervals – Newton forward and backward formula – Central Difference Interpolation formula – Gauss forward and backward formula – Stirling’s formula – Bessel’s Formula - Numerical differentiation: Maximum and minimum values of a tabulated function. Numerical Integration: Trapezoidal Rule – Simpson’s Rule – Numerical double Integration.

**UNIT IV: Basic Distribution**

Binominal distribution – Poisson distribution – Normal distribution – Properties and Applications.

**UNIT V: Correlation and Regression**

Correlation Coefficient – Rank correlation coefficient of determination – Linear regression –Method of least squares – Fitting of the curve of the form  $ax+b$ ,  $ax^2+bx+c$ ,  $ab^x$  and  $ax^b$  – Multiple and partial correlation (3-variable only).

**TEXT BOOK:**

1. **P. Kandasamy, K. Thilagavathy, K. Gunavathi**, “*Numerical Methods*”, 3<sup>rd</sup> Edition, S. Chand, 2006.
2. **S.C. Gupta and V.K. Kapoor**, “*Fundamentals of Mathematical Statistics*”, Sultan Chand & Sons, 1994.

UNIT	Chapter(s)	Sections
I	3 of [1]	3.1 to 3.4
II	4 of [1]	4.1 to 4.4, 4.8
III	8, 9 of [1]	8.1 to 8.8, 9.1 to 9.16
IV	7 of [2]	7.1 to 7.4
V	10 of [2]	10.1 to 10.7

**BOOKS FOR SUPPLEMENTARY READING AND REFERENCES:**

1. **S.Kalavathy**, “*Numerical Methods*”, Vijay Nicole, Chennai, 2004.
2. **S.S. Sastry**, “*Introductory Methods of Numerical Analysis*”, Prentice Hall of India, Pvt Ltd., 1995.

**LEARNING OUTCOMES:**

- After successful completion of the course, the students will be able to apply these concepts to solve algebraic and transcendental equations, system of linear equations, evaluate derivatives and integrals using numerical techniques. Further, students will be able to analyze the given data with the help of the above statistical tools.

<b>18UPMAT1S04</b>	<b>DISCRETE MATHEMATICS</b>	L	T	P	C
		3	0	0	3

**OBJECTIVE:**

The focus of the module is on basic mathematical concepts in discrete mathematics and on applications of discrete mathematics.

**UNIT – I: Mathematical logic**

Statements and Notations – Connectives – Negation – Conjunction – Disjunction – Statement formulas and Truth table – Conditional and Bi- conditional – well formed formulas - Tautologies.

**UNIT – II: Mathematical logic (Contd...)**

Normal forms – Disjunctive Normal forms – Conjunctive Normal forms – Principal Disjunctive Normal forms – Principal conjunctive normal forms – ordering and uniqueness of normal forms – the theory of inference for the statement calculus – validity using truth tables – Rules of inference.

**UNIT – III: Predicate Calculus**

The predicate calculus – Predicates – The Statements function, Variables and quantifiers – Predicate formulas – Free and bound variables – The universe of discourse – inference theory of the predicate calculus – Valid formulas and Equivalence – some valid formulas over finite Universes – Special valid formulas involving quantifiers – Theory of inference for the predicate calculus.

**UNIT – IV: Relations and ordering**

Relations – Properties of binary relation in a set – Partial ordering – Partially ordered set: Representation and Associated terminology – Functions – Definition and introduction – Composition of functions – inverse functions – Natural numbers – Peano axioms – Mathematical Induction.

**UNIT – V: Lattices and Boolean Algebra**

Lattices partially ordered sets: Definition and Examples – Some properties of Lattices. Boolean Algebra: Definition and example – Sub algebra, Direct Product and homomorphism – Boolean Functions – Boolean forms and free Boolean algebra – values of Boolean expression and Boolean functions.

**TEXT BOOK**

**J.P. Trembly**, and **R. Manohar**, “*Discrete Mathematical Structure with Applications to Computer Science*”, Tata McGraw Hill, 2001.

**REFERENCE BOOK**

**Dr. M.K.Sen** and **Dr. B.C.Charraborthy**, “*Introduction to Discrete Mathematics*”, Arunabha Sen Books & Allied Pvt. Ltd., 8/1 Chintamoni Das Lane, Kolkata – 700009, Reprinted in 2016.

<b>UNIT</b>	<b>Chapter(s)</b>	<b>Sections</b>
<b>I</b>	<b>1</b>	<b>1.1, 1.2.1 to 1.2.4, 1.2.6 to 1.2.8</b>
<b>II</b>	<b>1</b>	<b>1.3.1 to 1.3.5, 1.4.1 to 1.4.2</b>
<b>III</b>	<b>1</b>	<b>1.6.1 to 1.6.4</b>
<b>IV</b>	<b>2</b>	<b>2.3.1, 2.3.2, 2.3.8, 2.3.9, 2.4.1, 2.4.3, 2.5.1</b>
<b>V</b>	<b>4</b>	<b>4.1.1, 4.1.2, 4.2.1, 4.2.2, 4.3.1, 4.3.2</b>

**LEARNING OUTCOMES:** Students completing this course will be able

- to express a logic sentence in terms of predicates, quantifiers, and logical connectives.
- to apply the rules of inference and methods of proof including direct and indirect proof forms, proof by contradiction, and mathematical induction.
- to evaluate Boolean functions and simplify expressions using the properties of Boolean algebra.

\*\*\*\*\*